

HW3: Combinational Logic and Components

1. **Turn K-maps back into a truth table:** For each of the following, find the truth tables corresponding to the functions defined by the K-maps:

(a)

		A	
		0	1
BC	00	1	1
	01	1	0
	11	1	1
	10	0	0

A	B	C	output
0	0	0	1
0	0	1	1
0	1	0	0
0	1	1	1
1	0	0	1
1	0	1	0
1	1	0	0
1	1	1	1

(b)

		AB			
		00	01	11	10
CD	00	1	0	0	1
	01	1	0	0	1
	11	1	1	1	1
	10	0	1	0	0

A	B	C	D	output
0	0	0	0	1
0	0	0	1	1
0	0	1	0	0
0	0	1	1	1
0	1	0	0	0
0	1	0	1	0
0	1	1	0	1
0	1	1	1	1
1	0	0	0	0
1	0	0	1	0
1	0	1	0	0
1	0	1	1	0
1	1	0	0	0
1	1	0	1	0
1	1	1	0	1
1	1	1	1	1

2. Obtain an MSOP and an MPOS for each of the following functions defined by the K-maps:

(a)

MSOP: $\bar{A}\bar{B} + BC + AB$
 MPOS: $(\bar{A}+B) \cdot (A+\bar{B}+C)$

		A	
		0	1
BC	00	1	0
	01	1	0
	11	1	1
	10	0	1

Handwritten annotations for (a):
 - Red circle around (0,0) and (0,1) labeled $\bar{A}\bar{B}$
 - Blue circle around (0,0) and (0,1) labeled $(\bar{A}+B)$
 - Red circle around (1,1) and (1,0) labeled BC
 - Blue circle around (1,0) labeled AB
 - Blue circle around (1,0) labeled $(A+\bar{B}+C)$
 - Orange text: $A=C, B=D, C=A, D=B$

(b)

MSOP: $AB + \bar{A}\bar{C}D + A\bar{C}\bar{D}$
 MPOS: $(A+\bar{C})(A+D)(B+\bar{C})(\bar{A}+B+\bar{D})$

		AB			
		00	01	11	10
CD	00	0	0	1	1
	01	1	1	1	0
	11	0	0	1	0
	10	0	0	1	0

Handwritten annotations for (b):
 - Red circle around (0,1) and (0,0) labeled $\bar{A}\bar{C}D$
 - Red circle around (0,1) and (0,0) labeled $A\bar{C}\bar{D}$
 - Blue circle around (0,1) and (0,0) labeled $(A+D)$
 - Red circle around (1,1) and (1,0) labeled AB
 - Blue circle around (1,1) and (1,0) labeled $(\bar{A}+B+\bar{D})$
 - Blue circle around (1,1) and (1,0) labeled $(A+\bar{C})$
 - Blue circle around (1,1) and (1,0) labeled $(B+\bar{C})$

3. Simplifying K-Maps with Don't Care values: For each of the following K-maps:

- Find the MSOP expression (show groupings)
- Find the MPOS expression (show groupings)
- Are your solutions unique, or are there other minimum expressions?
- Does the MPOS = MSOP?

(a)

	ab			
cd	00	01	11	10
00	X	0	0	1
01	1	0	0	X
11	0	X	0	1
10	0	0	0	1

$A\bar{B}$
 $\bar{C}\bar{B}$

MSOP = $A\bar{B} + \bar{C}\bar{B}$

$A + \bar{C}$
 \bar{B}

MPOS = $(A + \bar{C})(\bar{B})$
 $= A\bar{B} + \bar{C}\bar{B}$

\Rightarrow MSOP = MPOS

unique, no more non-redundant grouping w/ less literals

(b)

	ab			
cd	00	01	11	10
00	1	X	0	1
01	1	1	1	0
11	0	0	X	0
10	X	0	1	1

creates non-unique term

$B\bar{C}D/ABD$
 $\bar{A}\bar{C}$
 $\bar{B}\bar{D}$
 $ABC/AC\bar{D}$

MSOP = $A\bar{C} + \bar{B}\bar{D} + ABC/AC\bar{D} + B\bar{C}D/ABD$
 or any similar answer

$\bar{A} + B + \bar{D}$
 $A + \bar{C}$
 $\bar{B} + C + D$

MPOS = $(\bar{A} + B + \bar{D})(A + \bar{C})(\bar{B} + C + D)$

non-unique, MPOS \neq MSOP

4. Adder design:

(a) Determine the complete truth table for a 2-bit adder (4 inputs A1, A0, B1, B0), and derive equations for both the sum (S1, S0) and the carry (Cout) outputs (no need to simplify).

A	B	A ₁	A ₀	B ₁	B ₀	Cout	S ₁	S ₀	result
0	0	0	0	0	0	0	0	0	0
0	1	0	0	0	1	0	0	1	1
0	2	0	0	1	0	0	1	0	2
0	3	0	0	1	1	0	1	1	3
1	0	0	1	0	0	0	0	1	1
1	1	0	1	0	1	0	1	0	2
1	2	0	1	1	0	0	1	0	3
1	3	0	1	1	1	1	0	0	4
2	0	1	0	0	0	0	1	0	2
2	1	1	0	0	1	0	1	1	3
2	2	1	0	1	0	1	0	0	4
2	3	1	0	1	1	1	0	1	5
3	0	1	1	0	0	0	1	0	3
3	1	1	1	0	1	0	0	1	4
3	2	1	1	1	0	1	0	1	5
3	3	1	1	1	1	1	1	0	6

$$C_{out} = A_1 A_0 B_1 B_0 + A_1 \bar{A}_0 B_1 \bar{B}_0 + A_1 \bar{A}_0 B_1 B_0 + A_1 A_0 \bar{B}_1 B_0 + A_1 A_0 B_1 B_0$$

$$S_1 = \bar{A}_1 \bar{A}_0 B_1 \bar{B}_0 + \bar{A}_1 \bar{A}_0 B_1 B_0 + \bar{A}_1 A_0 \bar{B}_1 B_0 + \bar{A}_1 A_0 B_1 \bar{B}_0 + A_1 \bar{A}_0 \bar{B}_1 B_0 + A_1 A_0 \bar{B}_1 \bar{B}_0 + A_1 \bar{A}_0 B_1 B_0 + A_1 A_0 B_1 B_0$$

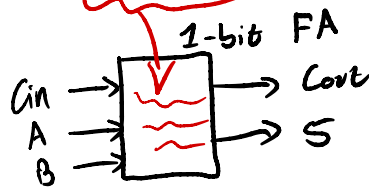
$$S_0 = \bar{A}_1 \bar{A}_0 \bar{B}_1 B_0 + \bar{A}_1 \bar{A}_0 B_1 B_0 + \bar{A}_1 A_0 \bar{B}_1 \bar{B}_0 + A_1 \bar{A}_0 \bar{B}_1 B_0 + A_1 \bar{A}_0 B_1 B_0 + A_1 A_0 \bar{B}_1 \bar{B}_0 + A_1 A_0 B_1 \bar{B}_0$$

(b) Design a 1-bit full adder (3 inputs A, B, Cin) (create a truth table, and show the equations) and use this component to create the equivalent of a 2-bit adder by chaining two components together. The overall circuit you create should have inputs (A1, A0, B1, B0, Cin), and outputs (S1, S0, Cout).

Cin	A	B	Cout	S
0	0	0	0	0
0	0	1	0	1
0	1	0	0	1
0	1	1	1	0
1	0	0	1	0
1	0	1	1	1
1	1	0	1	1
1	1	1	1	0

$$C_{out} = AB + C_{in} B + C_{in} A$$

$$S = \bar{C}_{in} \bar{A} B + \bar{C}_{in} A \bar{B} + C_{in} \bar{A} \bar{B} + C_{in} AB$$



2 bit full adder:

